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THE MULTIDIMENSIONALITY OF MUSICAL PITCH by Patrick Yves Berger

Two recurring concepts in the areas of sound healing and metaphysics are structure and flow. With aspects of sound healing trying to bridge the realms of music and metaphysics, this paper tries to explore the multidimensionality of pitch in terms of structure and flow.

The structure of pitch

Pitch is a perceptual property that allows the ordering of sounds on a frequency-related scale[1].

The concept of pitch multidimensionality can be best explained through the introduction of two key characteristics of musical pitch: pitch chroma and pitch height.

Pitch chroma can be defined as the distinctive quality of a specific tone, separating it from the rest of the tones within an octave[2].
It describes perceptual 'differences'/'distances' of pitches within an octave and the perceptual sameness of pitches separated by one or more full octaves. It is reflected in the fact that the different note names (e.g. C, D, E, F, G, A, B, C, D,...) repeat periodically for every 2/1 increase in frequency (i.e. every octave) with the addition of a subscript (e.g. C4) to indicate how high or low this pitch is relative to some reference pitch. In other words, a numeric subscript difference between two notes that share the same pitch chroma (e.g. C4 vs. C5) reflects a pitch height difference of one or more octaves between two notes.

Pitch height is a term describing the perceptual 'highness' or 'lowness' of a pitch and is related to frequency[3].
The intervals A3 (220Hz) - A4 (440Hz) and A4 (440Hz) - A5 (880Hz) are both octaves, with the three notes (A3, A4, A5) having the same pitch chroma but different pitch heights. In terms of their pitch height, octaves are equidistant perceptually although they are not equidistant in terms of Hz because, as indicated by Fechner's psychophysical law, perceptual and physical magnitudes of stimuli are related logarithmically (i.e. Hertz versus Pitch; Intensity in W/m2 versus Loudness in phons or sones; etc.).
Notes represented by different names are different in both pitch chroma and pitch height. Within a single octave, the pitch raises in accordance with the note name (C3 is lower than D3, which is lower than E3, etc.).

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[1] http://en.wikipedia.org/wiki/Pitch_(music)
[2] http://acousticslab.org/psychoacoustics/PMFiles/Module05.htm
[3] http://acousticslab.org/psychoacoustics/PMFiles/Module05.htm

The flow of pitch

Having explored the two basic characteristics underlying pitch, it is important to highlight that we are dealing with three dimensions within these concepts: one dimension within pitch height (frequency) and two dimensions within pitch chroma (separating pitches within an octave & linking pitches an octave apart)[4].

As a general rule, a variable is uni-dimensional if all its values can fit on a single straight line. If for example A, B, & C represent 3 values of a variable as points in space then:
a) if |AB| + |BC| = |AC| then the variable is uni-dimensional
b) if |AB| + |BC| ≠ |AC| then the variable is multidimensional

The base for this pitch multidimensionality can be visualized in our musical scale, which is based on the circular configuration in Figure 1, the chromatic scale.

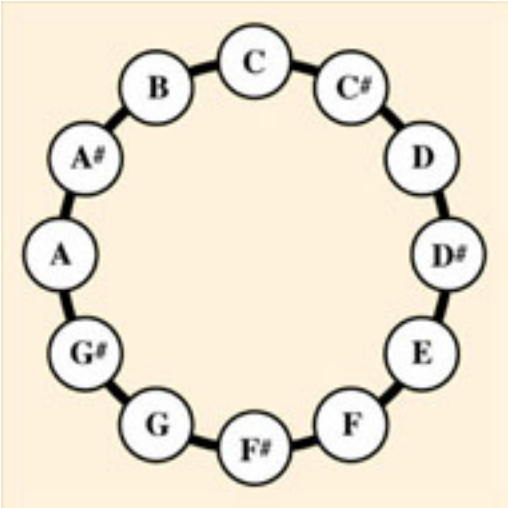

Figure 1 The chromatic scale

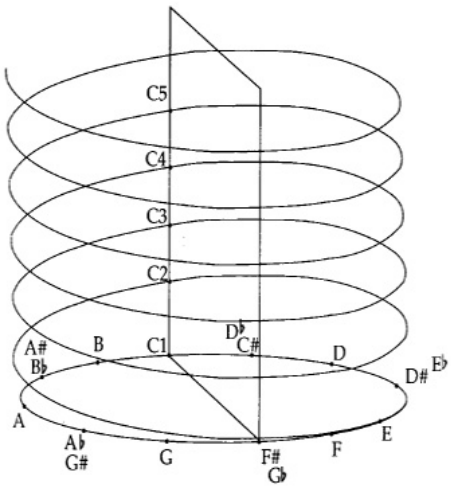

Figure 2 – The pitch helix

The western chromatic scale breaks the octave down into 12 different pitch chromas. The pitch chroma dimensions represent circularity in pitch perception. Due to this circularity, although the octave interval represents the largest physical distance (within a single octave) in terms of pitch height, it represents the smallest perceptual distance in terms of pitch chroma.

Pitch perception therefore wraps on the octave, with scales defining sets of different pitch chromas that repeat at different pitch heights for each new octave. This results in what may be called a "pitch helix" (Figure 2).

[4] http://acousticslab.org/psychoacoustics/PMFiles/Module05.htm

An often-used analogy for the perceptual circularity of pitch is the figure of the *impossible staircase*, similar to one originally published by Penrose and Penrose in 1958[5]. Since each stair that is one step clockwise from its neighbor is also one step downward, the staircase appears to be eternally descending. Our perceptual system insists on this interpretation, even though it cannot be correct.

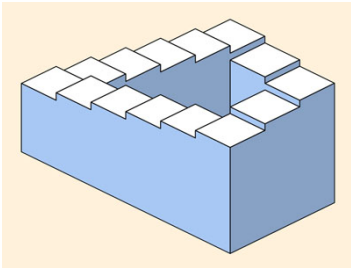

Figure 3 The impossible staircase

The impact of synthetic geometry to the multidimensionality of pitch

The basis of using a circle to define our musical system can be compared to the isoperimetric theorem applied by German mathematician, astronomer and astrologer Johannes Kepler (1571–1630)[6].

This theorem states that among all closed curves having a given parameter, the circle is the unique curve that encloses the greatest area. Circular action is the maximally efficient form of action in visible space and Kepler reasoned that if circular action reflects uniquely the creative process of the universe, then the form of everything which exists—of atoms and molecules, of the solar system, and the musical system—must be constructible using nothing but circular action, a procedure called synthetic geometry.

The procedure of "synthetic geometry" was taken further by Carl Friedrich Gauss at the beginning of the nineteenth century. Gauss introduced conical spiral action, instead of mere circular action, as the basis for synthetic geometry. Spiral action combines the isoperimetric principle of the circle with the principle of growth expressed by the Golden Section.

In elementary geometry, the Golden Section arises as the ratio between the side and the diagonal of a regular pentagon (see Figure 4). The Golden Section naturally forms what we call a self-similar geometric series—a growth process in which each stage forms a Golden Section ratio with the preceding one. Already before Leonardo da Vinci, Leonardo Pisano (also called Fibonacci) demonstrated that the growth of populations of living organisms always follows a

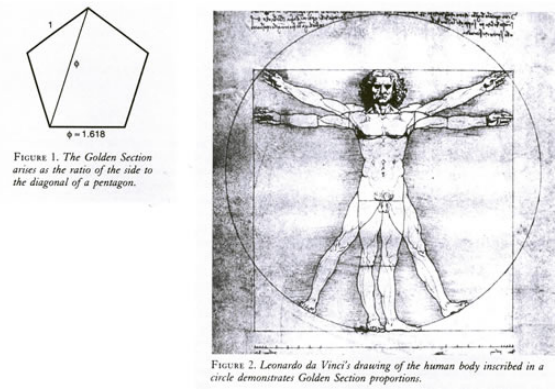

Figure 4 The Golden Section

[5] http://deutsch.ucsd.edu/psychology/pages.php?i=213
[6] http://en.wikipedia.org/wiki/Johannes_Kepler

series derived from the Golden Section. In extensive morphological studies, Leonardo da Vinci showed that the Golden Section is the essential characteristic of construction of all living forms.

Since music is the product of the human voice and human mind—i.e., of living processes—therefore, everything in music must be coherent with the Golden Section[7].

A practical example

According to Jonathan Tennenbaum, the conical spiral action introduced by Carl Friedrich Gauss, can be demonstrated through the singing dynamics in bel canto, which is a sophisticated model of singing that was evolving in early 17th-century Italy among practitioners of operatic and sacred music[8]:

Tennenbaum describes having a soprano sing a scale upward, starting at a middle C, i.e. 256 Hz. As the frequency increases, so does the intensity of the sound produced. The more precise term for this intensity is "energy flux density". This energy refers to the total response of the body to the making of sound; to a dynamic relationship between the breathing-in muscles and the breathing-out muscles known as the breath support mechanism; to the amount of breath pressure delivered to the vocal folds and their resistance to that pressure; and to the dynamic level of the sound. In summary, as one sings higher, one must use more energy; as one sings lower, one must use less[9]. But this increase is not merely linear extension, not merely increase in scalar magnitude. As the singer sings upward, two important events occur:

1) The singer must make a register shift, at F-sharp, in order to maintain the "isoperimetric", least-action form of bel canto tone.

A vocal register (Figure 5) is a particular series of tones in the human voice that are produced by one particular vibratory pattern of the vocal folds and therefore possess a common quality[10].

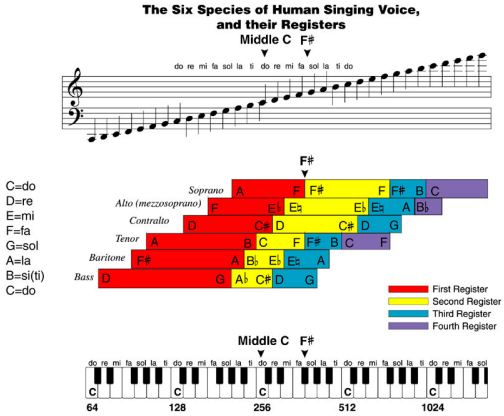

Figure 5 The vocal register

The register shift divides the scale exactly at the geometrical-mean or halfway point in the cycle of conical spiral action. The same process repeats in the next-higher octave, where the shift from second to third register of the soprano comes once again at F-sharp, the geometric mean.

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[7] http://www.schillerinstitute.org/fid_91-96/fid_911_jbt_tune.html
[8] http://en.wikipedia.org/wiki/Bel_canto
[9] https://en.wikipedia.org/wiki/Singing
[10] http://en.wikipedia.org/wiki/Vocal_register

The bel canto shift is a physical event of fundamental importance, and not merely a technical question for the voice. In physical terms, the register shift constitutes a singularity, a nonlinear phase change comparable to the transformation from ice to water or water to steam. Therefore the emphasis here is on the non-linearity of movement.

2) The second event occurs upon arrival at the octave, C=512. According to Tennenbaum, one can hear very clearly, that one cycle of action has been completed, like a 360° rotation and it proves that there is a rotational component of action to increase the frequency or energy flux density[11].

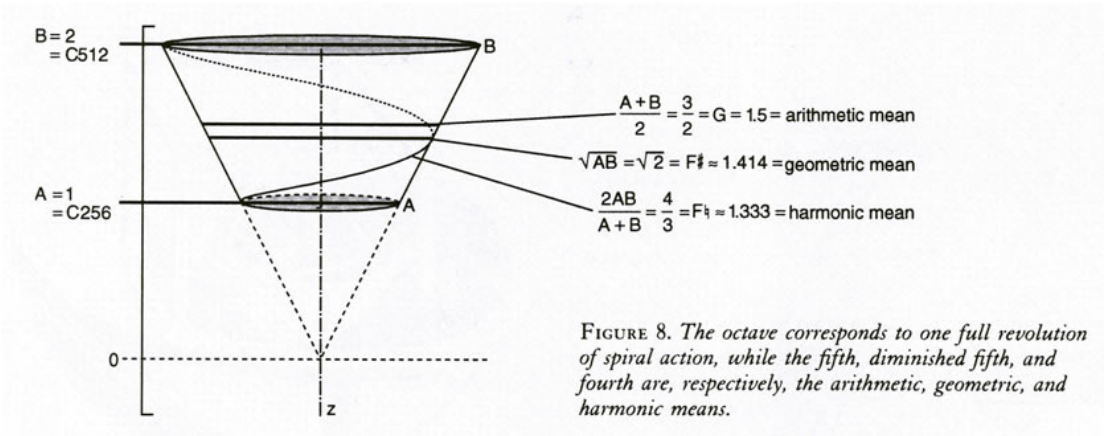

FIGURE 8. *The octave corresponds to one full revolution of spiral action, while the fifth, diminished fifth, and fourth are, respectively, the arithmetic, geometric, and harmonic means.*

Figure 6 The conical spiral

The true geometry of the singer's action is therefore most simply represented by spiral action upward on a cone. In Figure 6, the cone's axis represents frequency. Each circular cross-section of the cone represents a bel canto musical tone. The spiral makes one complete rotation in passing from C=256 to C=512, and one more cycle would bring it from C=512 to the next higher octave, C=1024. Thus, the interval of an octave corresponds to one complete 360° cycle of conical spiral action.

According to Tennenbaum, not only the octave, but all musical intervals, correspond to specific angles on conical-spiral action. This is most clearly seen if one projects the conical spiral onto a plane perpendicular to the axis (see Figure 7). If one divides a full 360° rotation into twelve equal angles, then each such (30°) displacement corresponds to a semitone interval in frequency. The radial lengths defined by the spiral at the indicated twelve angles are exactly proportional to the frequencies of the equal-tempered musical scale. The interval of a fifth corresponds to rotation through 7/12 of the circle, or 210°. The interval of a minor third

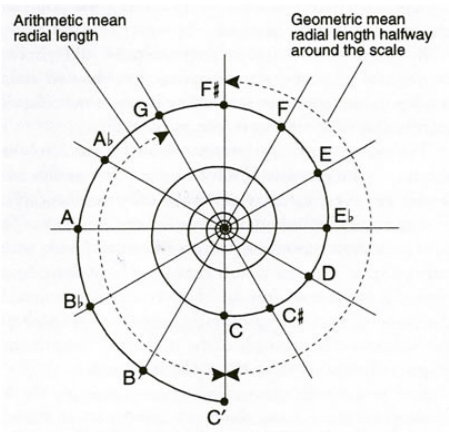

FIGURE 9. *The self-similar conical spiral projected onto a plane, showing the intervals of the equal-tempered scale.*

Figure 7 Conical spiral intervals

[11] http://www.schillerinstitute.org/fid_91-96/fid_911_jbt_tune.html

corresponds to a right angle, and so forth.

Most important, the halfway point of the full cycle starting at C, is F-sharp, the diminished fifth from C, or the interval once known as the "devil's interval." In terms of geometrical proportion, this F-sharp is located as the geometric mean of C=256 and its octave, C=512.

The multidimensionality of pitch: Helix or Vortex?

Although this paper has highlighted the multidimensionality of pitch, future focus can be directed to further combinations of structure and flow in terms of psychophysical laws and musical temperaments. As Tennenbaum comments:

“The equal-tempered system is only an approximation of a rigorous well-tempered system whose details have yet to be fully elaborated. Nevertheless, the indicated construction identifies the frequency regions and angular displacements within which the well-tempered values are to be defined.”